QUESTION 1

Let:

A = The length of the smallest angle bisector inside a triangle with side lengths 8, 12, and 16

B = Given an equilateral triangle with side length 5, a square of side length 6, and a circle of radius 3, let B be the area of the shape with the greatest area.

C = A square is inscribed in a circle of radius 10. A circle is then inscribed in this square, and a second square is inscribed in this circle, continuing this pattern forever. Let C be the area of the 10th largest square.

D = The sum of all the altitudes in a triangle with side lengths 12, 18, and 24.

Find $10A + B + 32C + 12D$

QUESTION 2

Let:

A = The negative y-intercept of the circle that contains the points (-6, 5), (-3, -4), and (2, 1).

B = The area of a tangential quadrilateral ABCD with $AB = 10$, a perimeter of 36, and an incircle with area $\frac{1600}{81}\pi$

C = The inradius of an equilateral triangle with an area of $144\sqrt{3}$

D = The total surface area of the figure composed of the locus of points that are 7 units away from a line segment of length 7

Find $\frac{A^2 \cdot B}{3C \cdot D}$

QUESTION 3

Let:

A = The length of HI in trapezoid $ABCD$, given that $AB = 3CD - 8$, $7CD = 3AB + 12$, and HI is parallel to the bases (AB and CD) and goes through the intersection point of segments AC and BD

B = Circle X has a radius of 4 with arc $\overset{\frown}{AB}$ of length π. Let B the square of the length of chord AB.

C = the length of BE given that for rectangle $ABCD$, $AE = 17$, $CE = 14$, $DE = 9$, and point E lies inside of the rectangle

D = There exists a cyclic quadrilareral ABCD inscribed in a circle of radius 15. Find the length of $\overset{\frown}{ADC}$, given $\angle ADC = 72^\circ$.

Find $\dfrac{24A\pi}{D} + C^2 + B$

QUESTION 4

The following statements have point values indicated by the numbers within the parentheses by each statement. Starting with 0, add the points of every true statement, and subtract the points of every false statement.

(21) Brahmagupta's theorem is used to find the area of a tangential quadrilateral.
(42) The two pairs of opposite sides in a tangential quadrilateral add up to the same total length.
(1337) Heron's formula is used to find the area of a triangle.
(-3) Ptolemy's theorem states that given a cyclic quadrilateral ABCD
with sides of length a,b,c, and d respectively, and diagonals of length e and f, $a \times c + b \times d = e \times f$.
(-1) Skew lines can never intersect.
(-4) Pick's theorem states that the area of a polygon taken on a grid is equal to $b + \frac{i}{2} - 1$ where b is the number of lattice points on the boundary of the polygon and i is the number of lattice points contained in the polygon's interior.
(-5) Ceva's theorem relates the way in which two concurrent cevians divide the respective sides of a triangle.

What is the final number of points?

QUESTION 5

Let:

A = The length of BA, given that triangle BAD is similar to triangle CRY, $AD = 34$, $CR = 12$, and $RY = 16$

B = The number of complete revolutions it would take for a circular wheel with radius $5\sqrt{2}$ ft. to travel a mile (Let $\pi = \frac{22}{7}$)

C = The y-coordinate of the centroid of an isosceles triangle with two vertices at the coordinates (7,2) and (13,-4), and given that the sum of the coordinates of the centroid is a single digit multiple of 3 and the triangle has an area of 54

D = The volume of a frustum with radii of lengths 18 and 34 and a slant height of 20.

Find $4A \div 3C + \frac{B}{2} + \frac{D}{16}$

QUESTION 6

Rayyan just bought a castle and is currently renovating it. He is trying to determine which tile he should buy so that he gets the best deal (most area per dollar). He has 5 options: R, A, Y, O, and N.

- Option R is a tile in the shape of a regular hexagon with side length 6 that costs \$7.
- Option A is a tile in the shape of an equilateral triangle with an angle bisector of length $4\sqrt{3}$ and costs \$2.
- Option Y is a tile in the shape of a circle represented by the equation $x^2 + y^2 - 54x - 84y + 2477 = 0$ that costs \$4.
- Option O is a tile in the shape of an octagon with side length 2 that costs \$1.50.
- Option N is a tile in the shape of an isosceles trapezoid with diagonal of length 17, all integer side lengths, and a perimeter of 50 that costs \$8.50.

What is the list of the 5 options in order from the best to worst deal? (Ex. Express in the form "A, B, C, D, E")

QUESTION 7

Let:

A = The slope of the equation of the line that the altitude from point B to AC in triangle ABC lies on given that point B is (7,6), point A is (15,6), and point C is (7,21).

B = The smallest angle that can be drawn using any three vertices on a regular octagon

C = The number of intersections of diagonals in the interior of a regular heptagon

D = Given the graph $x^2 + y^2 - 8y - 26x - 299 = 0$, find the sum of the abcissa of the center and the radius.

Find $\dfrac{A \times D}{B} + C$

QUESTION 8

The following statements have point values indicated by the numbers within the parentheses by each statement. Starting with 1, multiply the points of every true statement, and multiply the reciprocal of the points of every false statement.

(11) The orthocenter of a triangle is the location where all of that triangle's altitudes meet.

(9) The circumcenter of a triangle is equidistant from every side.

(15) The centroid of a triangle divides each median in the ratio 2:3.

(25) A distinct triangle is always defined by knowing two sides and an included angle.

(55) Euler's Line of a triangle always contains its circumcenter, orthocenter, and incenter.

(27) The diagonals of a rhombus are perpendicular bisectors of each other.

Return the final value.

QUESTION 9

Let:

A = The sum of an interior angle and all exterior angles of a 32-agon.
B = The number of diagonals in a 32-gon.
C = The surface area of a right cone with radius 14 and slant height of 50.
D = The volume of a right cone with radius of 14 and slant height of 50.

What is $(A + B)\pi + C + D$ in terms of π?

QUESTION 10

Let:

A = The distance between Mihir and Shreyas, who are standing at points (15,6) and (27, -29) respectively.

B = The area of a triangle with points at: (2,14), (6,10), and (4,6)

C = The shortest distance Eric, who is currently at $(-24,8)$, needs to walk to get to the river for water, which is at $y = -\frac{4}{3}x + \frac{7}{12}$.

Rayyan is trying to prove his great knowledge to Sanjita by "reading her mind". Rayyan asks Sanjita to pick an even prime, Sanjita falls right into Rayyan's trap of picking the number 2, not knowing 2 is the only even prime. Rayyan then tells Sanjita to take the number in her brain and multiply it by 10, and then subtract by 3 until she gets to a prime number. Rayyan, however, thought the 7th smallest prime number was composite. Rayyan picks the number he thinks Sanjita ended up with but it turns out he is wrong.

D = The product of the number Sanjita ended up with and the number Rayyan thinks Sanjita ended up with.

Evaluate: $A + B + 4C + D$?

QUESTION 11

Let:

A = The length of JS, given regular octagon $BLJKRSTU$ has a side length of 17.

B = The length of the obtuse angle formed by a clock at 7:57.

C = The number of handshakes interchanged by 73 people given that all 73 people shake one another's hand exactly once.

D = The number of ways to arrange 13 beads (4 blue, 6 red, and 3 green) in a straight line.

Evaluate $A + B + C + D$

QUESTION 12

Rohan, Aniketh, Vishal and Vishnav are going on an adventure on a triangular island. On the Cartesian plane of the world the island's vertices are at: R (5,22), H (20,17), S (11,3).

- Rohan, at vertex R, wants to meet Vishnav, who is at the centroid of the island for lunch. Let A equal the shortest distance Rohan has to walk to meet Vishnav for lunch.
- Vishal was walking from vertex H directly toward vertex S, but halfway through his journey from vertex H to vertex S, he remembers he needs to go to the centroid for lunch so he rushes to meet Vishnav and Rohan. Let B equal the total distance Vishal walked from vertex H till he met Rohan and Vishnav.
- Aniketh went to Point T (15,18), instead of the centroid. Mistaken, he too rushes over to the centroid. Let C equal the distance Aniketh had to run to get to the centroid from T.

List A, B, C from greatest to least.

QUESTION 13

Let:

A = The volume of a cylinder with radius 21 and height of 72.

B = The volume of the figure resulting from placing a hemisphere on top of a cone with total height of 21 and a great circle of circumference 30π.

C = The radius of a cone which has the volume of A+B and a height of 6.

D = Find the slant length of a cone which has a volume of 45π and the same radius as a sphere with a volume of 36π.

Evaluate: $\dfrac{A+B}{C^2\pi} + \dfrac{D}{\sqrt{26}}$

QUESTION 14

Varun and Anirudh each have a spa in their backyard. Both go about filling their spas, each 3ft x 8ft x 8ft pool, in different ways.

A = Anirudh is stingy and decides that he can fill it on his own without paying for a company to fill it for him. Anirudh lives an eighth of a mile away from a river and decides to buy two buckets which each carry 5 gallons of water and they each cost 5 dollars (given a cubic foot is 7.5 gallons). He carries the two buckets back and forth to fill up his spa. If Anirudh walks 4mph and values 1 hour of his time at 20 dollars. Let A = the total amount spent my Anirudh if he includes the price of his buckets and the compensation for his time.

B = Varun is lazy and decides he can just pay someone to fill his spa for him. He decides to employ Scott's Spas. Scott charges 10 cents per gallon of water, and half of the total price of water to get the water poured into the pool.

Find $A + B$ in simplest rational terms and in terms of π.